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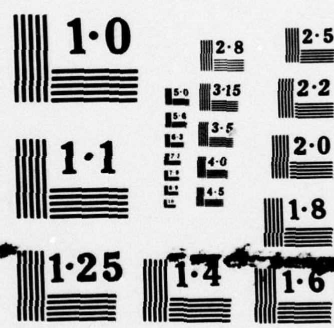

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FOREIGN TECHNOLOGY DIVISION

COMPRESSION OF A MAGNETIC FIELD BY A SHELL WITH CONSTANT CONDUCTIVITY

by

A. Ye. Kulago

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EDITED TRANSLATION

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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

0236

COMPRESSION OF A MAGNETIC FIELD BY A SHELL WITH CONSTANT CONDUCTIVITY

A. Ye. Kulago

§1. Statement of the problem and its solution by the integral transformation method. Ya. P. Terletskiy [1] was the first to suggest obtaining super-strong magnetic fields by compressing the field. Fields on the order of 10^7 G were obtained in experiments [2, 3] based on this method. The plane problem of the compression of a magnetic field without consideration of displacement currents was solved in [4-6]. The authors of [7] considered the diffusion of a magnetic field into a plate and a shell during the movement of the melting zone of the metal. The plane and axisymmetrical problems for a perfectly-conductive boundary were solved by I. M. Rutkevich with

consideration of the displacement currents [8]. The problem of the compression of a magnetic field by a cylindrical shell with finite conductivity is solved in this report without consideration of the displacement currents.

We will consider a cylindrical cavity whose initial radius R_0 is compressed at rate $\overleftarrow{V}$. Region D_2 is considered to be infinite. At the initial point in time $t = 0$ the magnetic field was homogeneous, $H = H_0$ in the cavity (region D_1), and $H = 0$ in the conductor (region D_2). The electrical field E was absent at $t = 0$, and at the center of the plane

$$E(0, t) = 0. \qquad (1)$$

In this case, the equations for H and E are:

$$\operatorname{rot}\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \quad \operatorname{div}\vec{H} = 0, \quad \operatorname{div}\vec{E} = 0, \qquad r \in D_1 + D_2. \qquad (2)$$

$$\operatorname{rot}\vec{H} = \frac{1}{c}\cdot\frac{\partial\vec{E}}{\partial t}, \qquad r \in D_1. \qquad (3)$$

$$\operatorname{rot}\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\cdot\frac{\partial\vec{E}}{\partial t}, \qquad (4)$$

$$\vec{j} = \sigma\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{H}\right], \qquad r \in D_2. \qquad (5)$$

We will use cylindrical coordinate system r, ϕ, z. The z-axis is the axis of symmetry of the given problem. Vector $\vec{H}$ will have the component $H_z(r, t)$, $\vec{E} = E_\varphi(r, t)$, and $\vec{v} = v_r = -v$. Without considering the displacement currents and keeping in mind that $\vec{E} = 0$, we can

consider the field H_z to be homogeneous in D_1, i.e., $H_z = H(t)$.

The conditions of the conjugation of the solutions on the interface of regions D_1 and D_2 will be

$$E|_{\gamma_-} = E|_{\gamma_+}, \quad H|_{\gamma_-} = H|_{\gamma_+},$$

i.e., the magnetic and electrical field on the interface is continuous. Integrating the first equation of system (2) from $r = 0$ to $r = R$, where $R = R_0 - \int_0^t v(u)\,du$, and considering (1), we will have

$$\frac{d(HR)}{dt} = -2cE - vH, \qquad (6)$$

where H is the field in cavity D_1 and E is the electrical field on the interface γ. Since the diffusion rate of the magnetic field is considerably higher than the rate of movement of the interface, we will solve the problem of the diffusion of a magnetic field into a stationary conductor, considering $R = R(t^*)$ to be constant at this stage. We will recalculate E and H for a mobile conductor from the known formulae:

$$\vec{H}' = \vec{H}, \quad \vec{E}' = \vec{E} + \frac{1}{c}[\vec{v} \times \vec{H}],$$

where the prime designates the moving coordinate system.

We will find E and H in D_2. In order to do this, we will use the integral Laplace transform:

$$H(r,p)=\int_0^\infty e^{-pt}H(r,t)\,dt,\quad E(r,p)=\int_0^\infty e^{-pt}E(r,t)\,dt.$$

Using equations (2) and (4) and disregarding the displacement currents, we will have

$$\frac{d^2H}{dr^2}+\frac{1}{r}\cdot\frac{dH}{dr}-\frac{4\pi\sigma}{c^2}pH=0. \qquad (7)$$

The solution to equation (7) will be

$$H(r,p)=A(p)Y_0(k,r),$$

where Y_0 is the Weber function with the zero subscript, approaching zero at infinity,

$$k^2=\frac{4\pi\sigma}{c^2}p.$$

We will find function A(p) from the boundary condition as follows. Let

$$H(r,p)\doteqdot\bar{H}(r,t),\quad A(p)\doteqdot\bar{A}(t).$$

Then [9]

$$\bar{H}(r,t)=\int_0^t\bar{A}(t-u)\bar{Y}_0(r,u)\,du.$$

and at r = R, on the boundary

$$\tilde{H}(R, t) = \int_0^t \tilde{\lambda}(t-u)\tilde{Y}_0(R, u)\,du.$$

Referring again to the representations, we will have

$$H(R, p) = A(p)Y_0(k, R),$$

where

$$H(R, p) \doteqdot H(t).$$

Then

$$H(r, p) = H(R, p)\frac{Y_0(kr)}{Y_0(kR)}.$$

Using the known operational analysis theorems [9]:

$$\frac{A(p)}{pB(p)} \doteqdot \frac{A(0)}{B(0)} + \sum_{k=1}^{\infty} \frac{A(p_k)}{p_k B'(p_k)} e^{p_k t},$$

$$p\Phi_1(p)\Phi_2(p) \doteqdot \frac{d}{dt}\int_0^t f_1(t-u) f_2(u)\,du,$$

where A(p), B(p) are polynomials for p, we will find

$$H(r,t)=\frac{d}{dt}\int_0^t H(u)\left[1+2\sum_{k=1}^{\infty}\frac{Y_0\left(q_k\frac{r}{R}\right)}{Y_1(q_k)\,q_k^2}\,e^{-\frac{q_k^2}{\alpha(t)}(t-u)}\right]du,$$

where q_k is the root of $Y_0(q)=0,\ kR=q=\sqrt{\frac{4\pi\sigma}{c^2}R^2}\,\sqrt{p}=\sqrt{\alpha}\sqrt{p},\ Y_1(q)=\frac{dY_0}{dq}$.

Here H(r, t) is the field in D_2 and H(t) is the field in D_1.

If we disregard the displacement currents, magnetic field H is the same for mobile and stationary conductors. The electrical field for a **stationary conductor** is equal to

$$E=\frac{c}{4\pi\sigma}\cdot\frac{\partial H}{\partial r}=-\frac{c}{2\pi\sigma}\cdot\frac{d}{dt}\int_0^t H(u)\sum_{k=1}^{\infty}\frac{Y_1\left(q_k\frac{r}{R}\right)}{q_kR(t)\,Y_1(q_k)}\,e^{-\frac{q_k^2}{\alpha(t)}(t-u)}du.$$

Then

$$E|_{\gamma}=\frac{c^2}{2\pi\sigma}\cdot\frac{d}{dt}\int_0^t H(u)\sum_{k=1}^{\infty}\frac{1}{R(t)\,q_k}\,e^{-\frac{q_k^2}{\alpha(t)}(t-u)}du. \qquad (8)$$

Using equations (5), (6) and (8), we will obtain the equation for the strength of the magnetic field in the cavity

$$\frac{d(RH)}{dt}=\frac{c^2}{\pi\sigma}\cdot\frac{d}{dt}\int_0^t\frac{H(u)}{R(t)}\sum_{k=1}^{\infty}\frac{1}{q_k}e^{-\frac{q_k^2}{a}(t-u)}du+vH. \quad (9)$$

Integrating equation (9), we will have

$$H(t)=\frac{c^2}{\pi\sigma}\cdot\frac{d}{dt}\int_0^t\frac{H(u)}{R^2(t)}\sum_{k=1}^{\infty}\frac{1}{q_k}e^{-\frac{q_k^2}{a(t)}(t-u)}du+\int_0^t\frac{v(u)}{R(t)}H(u)(du)+\frac{H_0R_0}{R(t)}. \quad (10)$$

Equation (10) is the integral Volterra equation.

§2. Uniform movement of interface. Limiting case of high conductivity. We know [10] the following asymptotics for the roots:

$$q_k\cong q_1+(k-1)\pi.$$

We will limit ourselves to one term of the series in equation (10) and we will consider large values of σ. We will assume that $R = R_0 = v_0t$. Here v_0 = const. Then equation (10) assumes the form

$$H(t)=\frac{c^2}{\pi q_1\sigma}\int_0^t\frac{H(u)}{R^2(t)}du+\int_0^t\frac{v_0}{R(t)}H(u)du+\frac{H_0R_0}{R(t)}. \quad (11)$$

Differentiating equation (11) by t and substituting $\int_0^t H(u)du$ from equation (11) in the expression obtained, we will have the first-order differential equation for H(t):

$$\frac{dH}{dt}=\frac{(c^2+\pi q_1\sigma v_0 R)^2+\pi q_1\sigma R\,(2v_0c^2+\pi q_1\sigma R v_0^2)}{\pi q_1\sigma R^2\,(c^2+\pi q_1\sigma v_0 R)}H-\frac{H_0R_0v_0c^2}{R^2\,(c^2+\pi q_1\sigma v_0 R)}, \qquad (12)$$

The solution to equation (12) will be

$$H=H_0\frac{(c^2+\pi q_1\sigma v_0 R)\,R_0^3}{(c^2+\pi q_1\sigma v_0 R_0)\,R^3}\left[2-\frac{R_0}{R}+\frac{\pi q_1\sigma v_0 R_0}{c^2}\ln\frac{R_0\,(c^2+\pi q_1\sigma v_0 R)}{R\,(c^2+\pi q_1\sigma v_0 R_0)}\right]e^{\frac{c^2(R_0-R)}{\pi q_1\sigma v_0 R R_0}}. \qquad (13)$$

If σ in equation (13) approaches infinity, we will have

$$H=H_0\frac{R_0^2}{R^2}. \qquad (14)$$

Here we use the relationship

$$\lim_{\sigma\to\infty}\frac{\pi q_1\sigma v_0 R_0}{c^2}\ln\frac{R_0\,(c^2+\pi q_1\sigma v_0 R)}{R\,(c^2+\pi q_1\sigma v_0 R_0)}=\frac{R_0-R}{R}.$$

Solution (14) is the solution for a perfect conductor [1]. The figure shows a comparison of solution (14) with solution (13). The value of σ_1 for copper at room temperature is used. The curve for σ_3 has a maximum. Thus, the solution is correct up to specific values of the compression of the cavity, whence it follows that the approximate equation (11) is only valid at sufficiently large values of σR.

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the study.

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NIIMekhaniki [Scientific Research Institute of Mechanics]

Summary

A. E. Kulago

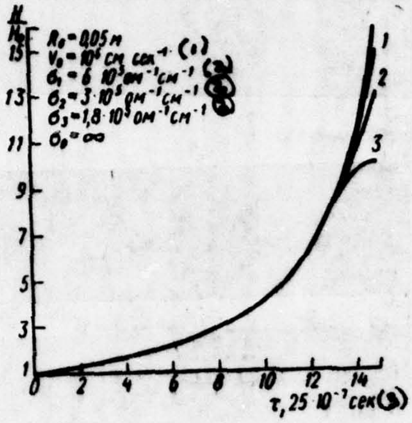
COMPRESSION OF A MAGNETIC FIELD BY A SHELL OF CONSTANT CONDUCTIVITY

Equations are obtained for a magnetic field compressed by a cylindrical shell of constant conductivity. Solutions are given in some particular cases.

Bibliography

1. Терлецкий Я. П. Получение сверхсильных полей путем быстрого сжатия проводящих оболочек. ЖЭТФ, 32, вып. 2, 1957.
2. Fowler C. M., Garn W. B., Caird R. S. Production of very high magnetic fields by implosion. «J. Appl. Phys.», 31, No. 3, 1960.
3. Сахаров А. Д. Взрывомагнитные генераторы. «Успехи физ. наук», 88, вып. 4, 1966.
4. Биченков Е. И. Влияние конечной проводимости на получение сильных магнитных полей путем быстрого обжатия проводящих оболочек. ПМТФ, № 6, 1964.
5. Paton A., Millar W. Compression of magnetic field between two semi-infinite slabs of constant conductivity. «J. Appl. Phys.», 35, No. 4, 1964.
6. Lehner G. Limitations on magnetic fields obtained by flux — compression. «Nuclear fusion», 4, 1964.
7. Семченко В. В., Степанов А. В. О диффузии импульсных сверхсильных магнитных полей. ПМТФ, № 1, 1969.
8. Руткевич И. М. Электромагнитное поле в сжимающейся полости. ПММ, 31, вып. 3, 1967.
9. Лаврентьев М. А., Шабат Б. В. Методы теории функций комплексного переменного. «Наука», М., 1965.
10. Ватсон Г. Н. Теория Бесселевых функций, ч. 1. М., ИЛ, 1949.

Figure. KEY: (1) $cm \cdot s^{-1}$. (2) $\Omega \cdot cm^{-1}$. (3) s.



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